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SUITABLE

ROYAL AIRCRAFT ESTABLISHMENT

Farnborough Hants

AN ELABORATION OF THE CRITERION FOR WING TORSIONAL STIFFNESS

BY

A.R. COLLAR, M.A.
E.G. BROADBENT, B.A.
AND
ELIZABETH B. PUTTICK

RECEIVED AUG 12 1947

WITH AN APPENDIX ON

THE EFFECT OF TAPER ON WING FLEXURE-TORSION FLUTTER SPEEDS

BY

E.G. BROADBENT, B.A.
AND
ELIZABETH B. PUTTICK

AIR DOCUMENTS DIVISION, T-2
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ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

An Elaboration of the Criterion for Wing Torsional Stiffness

by

A.R. Collar, M.A.
E.G. Broadbent, B.A.
and
Elizabeth B. Puttick.

with an appendix on

The Effect of Taper on Wing Flexure-torsion flutter speeds

by

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R.A.E. Ref: S.M.E.D.4/1854/E.G.B./125

SUMMARY

The paper describes briefly the work which led to the adoption of the present criterion for wing torsional stiffness and discusses the results obtained by later researches. It is shown that the critical flutter speed may be sensitive to parameters which do not appear in the existing criterion, among the most important of which are the position of the principal axis of inertia, and the wing taper. The paper puts forward a suggestion for a new criterion which includes the effect of these two parameters and also an allowance for compressibility.

The Appendix describes a fairly thorough investigation into the effects of taper, which includes also the variation of several other parameters.

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1 Introduction

The present criterion[1] for the torsional stiffness of wings has been established for nearly ten years; a brief history of its development is given in Para.2. It originated in a dimensional study of wing-aileron flutter and wing divergence; but in recent years it has assumed almost exclusively the function of a preventive of wing flexure-torsion flutter. In this role it has played a very important part in aircraft design.

In view of its extreme simplicity - the majority of the parameters affecting flexure-torsion flutter do not appear in the criterion - its uniform success as a flutter preventive is remarkable, and indeed suggests strongly that, in the many cases where the ignored parameters have had favourable values, the criterion must have been unnecessarily severe. However, until the last year or so this possibility has not been sufficiently important to warrant amendment of the criterion. Some such change might have been called for, had it not been that other factors affecting wing stiffness - in some cases design for strength, but more often considerations relating to rolling power - have themselves required a degree of stiffness of the same order as that defined by the criterion. Only very recently have designers had real difficulty in meeting the requirement of the criterion. This has been principally due to two factors, namely the increasing use of pronounced wing taper, and the necessity to make some provision (often of an arbitrary nature) for anticipated compressibility effects.

As regards taper, this is of particular importance, since in general the local torsional stiffness per unit span of modern wings falls off very rapidly towards the tip, often as the third or fourth power of the chord. In consequence, in the usual wing stiffness test (which provides the value of the stiffness appearing in the criterion) the rate of twist near the mid-aileron section is many times greater than that at the root. As a result, the designer finds that the most economical way of improving the overall stiffness is not by a distributed increase but by a pronounced increase near the midaileron section. Since this effect results from the use of taper, and since in fact it has been known for some years that taper is beneficial from the flutter viewpoint, a modification to the criterion to allow for this is obviously desirable. This is more particularly so since taper is beneficial also in regard to aileron reversal, and calculations relating to the latter take account of this fact; so that in comparison with the stiffness criterion, aileron reversal appears to be of less relative importance than is warranted by the true state of affairs.

Regarding the effect of Mach number, it is to be noted that practically all stiffness requirements now include a compressibility correction, and the absence of such a correction in the wing torsional stiffness requirement provides a major inconsistency. It is true that there is as yet practically no information on the effects of compressibility on flutter, but what information does exist suggests that the correction can be as large as that which would obtain if all the aerodynamic coefficients were increased by the Glauert factor, at least so long as shock waves are absent.

These considerations have provided additional impetus to the work of determining how the critical speed for wing flexure-torsion flutter depends on the principal parameters involved. The present report describes briefly the major steps in this work, and proposes a criterion which depends in a simple way on these parameters. In the event of the adoption of this new criterion, however, it will have to be remembered that it relates almost exclusively to the prevention of wing flexure-torsion flutter.

2 A Brief History of the Present Criterion

The following survey does not pretend to completeness, but is intended to present only the more important factors which have led to the present form of the criterion. This is (see for example A.P.970[1], Pt.5)

$$\frac{1}{V}\left(\frac{m_\theta}{dc_m^{\,2}}\right)^{\frac{1}{2}} > C, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where

V = design speed (ft./sec. E.A.S.)
m_θ = torsional stiffness of wing, measured between root and reference section (lb.ft./rad.)
d = distance from root to equivalent tip (ft.)
c_m = mean chord (ft.)

The quantity C is a number which depends on the reference section; it assumes different values for the mid-aileron and equivalent tip. Its value depends also on whether the wing carries engines or not; furthermore, if the wing is not carrying engines C may be a function of the wing density σ_w.

The form of the criterion (1) is due to Roxbee Cox[2]. His analysis was in the nature of a dimensional study*; it was assumed that in the families of wings considered the distributions of elasticity and inertia corresponded, and the distribution of chord with span was similar. It then emerged that, if the wing density was constant**, certain relations hold between the remaining parameters. Thus the wing torsion-aileron flutter speeds are connected by the relation

$$\frac{1}{V}\left(\frac{m_\theta}{sc_m^{\,2}}\right)^{\frac{1}{2}} = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Similarly, the flexure-aileron flutter speeds are related by

$$\frac{1}{V}\left(\frac{l_\phi}{s^3}\right)^{\frac{1}{2}} = \text{constant}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where in equations (2) and (3) s is the semi-span and l_ϕ is the wing flexural stiffness.

Roxbee Cox showed also[2] that equation (2) holds for the critical speed for wing divergence. To complete the study, he made a survey of the values of the criteria (2) and (3) achieved by a large number of aircraft. In view of the interest then attaching to wing density, which had been shown by Pugsley[3] to be important in relation to wing-aileron flutter***, he plotted these values against wing density. Though there was considerable scatter, the plot showed a general tendency for the criteria to increase with wing density.

---

* But the criterion itself is not non-dimensional; see remarks in Para.6.

** The wing density was not defined as such; it was defined as $/\rho$ where δ is the mass per unit area. This conception has given rise to the modern definition of the density σ_w.

*** This was true for the small values of wing density which were then common. For modern wings, which have much higher densities, density is much less important; see later.

The value of the criterion (2) was further underlined by studies of loss and reversal of aileron control; it was shown by Pugsley and Roxbee Cox[4] that in the evaluation of aileron power of a monoplane, the criterion (2) plays an important part.

The fact that the criterion (2) might be of great importance in the prevention of flexure-torsion flutter was first thrown into relief by the appearance of Kussner's flutter speed formula, which was described and discussed by Pugsley[5]. Kussner's formula states in effect that (within limited ranges of the parameters) flutter will be avoided provided

$$\frac{nc_m}{V} > k \qquad \text{...(4)}$$

where k is a constant and n is the frequency of the predominantly torsional mode of oscillation found in a ground resonance test. Now for families of wings of similar plan forms and having similar distributions of mass and elasticity, the frequency n will be proportional to the square root of the torsional stiffness m_θ divided by the torsional moment of inertia $\sigma_w s c_m^4$; hence (4) can be reduced to

$$\frac{1}{V}\left(\frac{m_\theta}{s c_m^2}\right)^{\frac{1}{2}} > k\sigma_w^{\frac{1}{2}} \qquad \text{.....................................(5)}$$

where k is a constant. This result agreed remarkably well with the findings of Roxbee Cox; and accordingly in a paper on wing stiffness Pugsley[6] proposed that criteria similar to (2) and (3) should be adopted as design criteria for aircraft. Pugsley here defined two reference sections, the mid-aileron and equivalent tip sections, and introduced the distance d from root to equivalent tip into the criteria in place of the semi-span s. He also pointed out that the torsional stiffness criterion is of more importance than the flexural criterion. The actual numerical values proposed were based on statistics of achieved criteria.

At about the same time, the criteria were given the status of recommended practice in A.D.T.N.19[7]; and shortly afterwards, the mandatory document A.D.M.374[8] appeared, which defined the torsional stiffness criterion substantially as it now appears in A.P.970[1]. A distinction was drawn between wings with wing engines and wings without engines. For the former, a constant value, independent of wing density, was required for the criterion; for the latter a linear variation between two given densities. This linear variation is an approximation to the parabolic relation given by (5) within the range of density defined.

3 Subsequent Theoretical Investigations of Flexure-torsion Flutter

As has already been indicated, the wing torsional stiffness criterion was not originally concerned with flexure-torsion flutter, except in so far as divergence can be regarded as a part of this question. However as its importance in the role of a preventive of flexure-torsion flutter became realized, the practice developed of expressing the results of researches into flexure-torsion flutter in terms of quantities closely allied to that of the criterion. The present paragraph sets out the main advances resulting from these researches; since they all relate to the same subject, it is convenient to describe them under author headings.

First, however, it should be noted that the wings of modern aircraft (or at least the military types) are very different from those of ten years ago in respect of wing density σ_W and of ratio of flexural to torsional stiffness r. As regards density the weight of wings has risen as a consequence of the usual adoption of all metal construction to meet the strength and stiffness demanded by modern high speeds; to obtain inertia relief more and more weight (such as fuel) has been distributed in the wings, and auxiliary masses such as ailerons and armament have become larger. In consequence σ_W has risen from the value 0.2 lb./cu.ft. common ten years ago to a value nearly ten times as great. Moreover, the essential factor in density considerations is not the absolute wing density but its ratio to the air density; and since on the average modern aircraft fly higher than those of ten years ago the average effective wing density has risen from this cause also.

The ratio r is defined in the light of the criteria (2) and (3) as

$$r = \left(\frac{\ell_\phi}{d^3}\right) / \left(\frac{m_\theta}{dc_m^2}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Ten years ago r commonly lay in the range between 5 and 10. In the intervening period, however, the average value of m_θ has risen at a far greater rate than ℓ_ϕ, while external bracing has largely disappeared, so that a good mean value for r nowadays is about unity.

3.1 Williams

The work of Williams[9] was carried out at an early date, some years before the criterion gained currency. The effects of variation of a large number of parameters were examined, and the principal conclusions were

(a) The ratio of torsional to flexural stiffness should be as large as possible,

(b) Small forward movements of the inertia axis can have powerful stabilizing effects.

(c) Rearward movement of the flexural axis (within the limits permitted by considerations of divergence) is favourable.

(d) Increase of wing density or decrease of air density is unfavourable.

(e) The effect of hysteresis is small.

Although the wing considered, being rectangular with σ_W = 0.2 lb./cu.ft., is unrespresentative of modern practice, the conclusions (a), (b), (c) and (e) still hold good.

3.2 Duncan and Lyon

In this investigation[10] a straight-tapered wing was assumed; the flexural axis was kept fixed at 0.3 chord from the leading edge, and the following parameters were varied: (a) the ratio of air density to wing density ε, (b) the ratio of flexural to torsional stiffness r (see equation (6)), (c) the position of the inertia axis, (d) the structural damping. The results are expressed in a form closely allied to the inverse of the torsional stiffness criterion and alternatively in the form of a Küssner coefficient. The principal conclusions are that in almost all cases

(a) For a given torsional stiffness, reduction of flexural stiffness is beneficial

(b) Increase in the torsional stiffness is beneficial

(c) Forward movement of the inertia axis produces a marked rise in critical speed.

(d) The influence of structural damping is not very marked.

(e) The value of the Küssner coefficient depends acutely on the position of the inertia axis.

(f) Increase in wing density lowers the critical flutter speed, but not indefinitely; an asymptotic condition is reached where the critical speed tends to a finite value for infinite wing density (this conclusion was not new, having been discussed earlier in relation to propeller flutter[11]).

The divergence speed was also discussed, and an Appendix drew attention to the difference between the stiffnesses appropriate to the flutter condition and those found in a stiffness test.

3.3 Duncan and Griffith

This was the first research[12] in which the wing taper was systematically varied. With this addition the work was in the main an extension of that described in 3.2; there was however, the difference that the inertia axis was kept fixed and the position of the flexural axis was varied. Throughout the work, the modes of deformation were kept constant. The conclusions additional to those of 3.2 are

(a) Increase in taper is beneficial

(b) Rearward movement of the flexural axis is beneficial.

Regarding (b) it may be noted that a comparison of the results of the two investigations[10,12], shows that rearward movement of the flexural axis is not as beneficial as forward movement of the inertia axis; this is of course consistent with the well known result that the optimum condition is that for which the inertia and flexural axes coincide with the axis of independence [13] (i.e. are near the aerodynamic centre).

3.4 Jahn and Buxton

This was a research[14] into the various methods of calculation of flutter speed adopted in Great Britain and other countries. The basic parameters employed were not stiffnesses but frequencies, and hence the results do not bear directly on the question of the stiffness criterion. However, one result of importance emerged, namely that the critical flutter speed is strongly influenced by the assumed mode of wing torsion; the effect of the mode of flexure is much less marked. Other conclusions of interest were that the addition of overtone modes (one in flexure and one in torsion) did not change the calculated critical speeds to any great extent, and that the most important aerodynamical actions by far are two stiffnesses, namely the rates of change of lift and pitching moment with incidence.

3.5 Buxton

This work[15] is to be reported shortly. In essence it is an extension of the work of Duncan and Griffith described in 3.3 to include variation of the position of the inertia axis; this variation to be given a wider range than in previous researches. Buxton derives from his results an emperical formula for flutter speed which shows that the position of the inertia axis is much more important than that of the flexural axis. He examines also the flutter of an elliptical wing, and finds that it has much the same critical speed as a straight-tapered wing of taper ratio about 0.6.

As in the work of Duncan and Griffith[12] the modes of deformation were not varied with the taper.

4 The present investigation

The work described in Para.3, and that of Duncan and Griffith in particular, had given a strong indication of the most important parameters in the problem of flexure-torsion flutter; and accordingly a simple extension of the torsional stiffness criterion was tentatively schemed. It was based on a diagram of the kind shown in Fig.1, which has been obtained from the results of Duncan and Griffith.

Three main curves are shown, A, B, and C. A is the parabolic variation of the criterion (1) with σ_W suggested by Pugsley; B is the actual variation of the criterion given in the requirements. The curve C is the value of the criterion for one of the wings considered by Duncan and Griffith at its critical flutter speed; i.e. V_c has been substituted for V in the criterion. This curve, described for simplicity as "standard" has a taper ratio k (tip chord divided by root chord) of 0.6 and a value of the stiffness ratio r of unity. Its flexural axis is at 0.3 chord and its inertia axis at 0.4 chord aft of the leading edge. It may be noted that the curve is roughly parabolic near $\sigma_W = 0$ (though in fact it does not pass through the origin); but that as σ_W is increased the slope of the curve soon becomes very small, and for values of σ_W greater than about 1.0 the ordinate is not very different from that of the asymptote corresponding to infinite wing density.

Suppose that, for example, this standard wing has a wing density of 1.2 lb./cu.ft. and a torsional stiffness such that, at its maximum diving speed of 500 ft./sec., the torsional stiffness criterion just has the required value of 0.04. Let the speed V appearing in the criterion (1) be regarded as a current variable. Then as the speed rises from a low value, the ordinate at $\sigma_W = 1.2$ drops, until at a speed of about 400 ft./sec. the parabolic curve A is reached. A further rise to 500 ft./sec. brings the ordinate down to curve B. A further rise brings the point down towards curve C, which is reached at a speed of about 800 ft./sec. - the critical speed. Thus the difference between curves B and C represents the safety margin between the maximum diving speed and the speed of onset of flutter.

The curves in the neighbourhood of C show the effects of variation of some of the relevant parameters. A change from h = 0.3 to h = 0.4 in the flexural axis position gives a slight improvement in the flutter condition. A change of r from 1.0 to 2.0 has a rather bigger worsening effect. The largest change shown is however, the improvement consequent on a change in the taper ratio k from 0.6 to 0.3.

Since the margins of stability are evidently considerably different with differing values of the parameters, a simple criterion, of the form

$$\frac{1}{V}\left(\frac{m_\theta}{d c_m^{\,2}}\right)^{\frac{1}{2}} > C\, f_1(k)\, f_2(r)$$

where f_1 and f_2 are appropriate functions would, it was thought, provide more uniform margins. However, this tentative criterion was not put forward, since further work suggested that some important effects were not included.

4.1 Secondary Effects of Taper

The Appendix to the present Report shows in detail the steps leading to the proposed new criterion, and gives the appropriate analysis. A brief description will therefore suffice here.

It was remarked in Para.3 that the assumed mode of torsion could have a pronounced effect on the critical flutter speed; but the investigations relating to taper assumed no variation of the mode of torsion with taper. Accordingly a new investigation was made to determine the theoretical mode of purely torsional oscillation in vacuo as a function of taper, variation of the distribution of stiffness and inertia with taper being chosen to represent average practical conditions. At the same time an investigation was put in hand to determine how far the stiffness appropriate to the actual mode of torsion assumed (corresponding to the mode of torsion in vacuo) differed from that found in a stiffness test. Here an important factor is the position of the reference section. This question had been discussed by Victory[16], but the results related to semi-rigid wings only; and it was concluded that the section at about 0.7 span from the root was the optimum. A note by Collar[17] had again drawn attention to the difference between the stiffness found in a stiffness test and that appropriate to a distributed load condition, and the question was discussed more fully in relation to aileron power by Collar and Broadbent[18] in a report from which the importance of the reference section may be deduced. It had of course been demonstrated at a very early date[11] that the difference between static and dynamic stiffness could be very large when the reference section for a flutter problem is taken at the tip.

The results of these two investigations were next embodied in further calculations of the critical flutter speed of a family of straight-tapered wings. The wing density chosen for these calculations was σ_W = 1.6 lb./cu.ft.; the principal variables assumed were r (stiffness ratio), k (taper ratio) and g (position of inertia axis as fraction of chord from leading edge). Some additional calculations were also made of the effect of h (position of flexural axis). The main results are shown in Fig.2, where the non-dimensional quantity

$$B = V_o \rho^{\frac{1}{2}} / \left(\frac{m_\theta}{dc^2}\right)_m^{\frac{1}{2}}$$

is plotted against r for various values of k and g.

4.2 An Expression for the Critical Speed

From Fig.2 the variation in critical flutter speed with the relevant parameters may at once be read off; and it is evident that we may write

$$\frac{1}{V_c} \left(\frac{m_\theta}{dc_m^2}\right)^{\frac{1}{2}} = C \rho_o^{\frac{1}{2}} F(k, g, r, h) \quad \text{.................(7)}$$

where V_c is an equivalent airspeed.

The determination of a really accurate function F would in general present considerable difficulty; fortunately however, quite good accuracy can be achieved by expressing F as a product of four factors each involving only one of the parameters. Moreover, within the practical range the four factors can be expressed as simple functions;

in fact, it will be found that the following expression gives curves very similar to those of Fig.2:

$$\frac{1}{V_c}\left(\frac{m_\theta}{dc^2_m}\right)^{\frac{1}{2}} = 0.9\,\rho_0^{\frac{1}{2}}\ \frac{(g - 0.1)\,(1.3 - h)}{(1 - 0.8k + 0.4k^2)\,(1 - 0.1r)} \quad .....(8)$$

4.3 The Effect of Compressibility

In all work described above no account has been taken of the effects of compressibility. Very little research on this question has in fact ever been carried out. Insofar as it is generally accepted that the usual Glauert correction[19] can be applied to quasi-static conditions below the critical Mach number, there is a case for such a correction, since divergence is part of the flutter problem. Moreover, the work of Frazer[20] on compressibility effects on flutter, supplemented by some unpublished work due to Jahn, has shown that in some circumstances the flutter speed can be modified by an amount equivalent to that of the Glauert correction. Accordingly it is proposed to introduce this correction into the criterion, subject to the limitation that for Mach numbers in excess of 0.8 only the correction appropriate to a Mach number of 0.8 need be applied.

4.4 Spanwise Variation of Mass Distribution

It is evident from (8) that almost the most important parameter in the flutter problem is the chordwise position of the mass centres of the sections, defined by g. A rearward movement of the centre of mass from 0.4 to 0.5 changes the relevant factor in the ratio 4 to 3, and thus implies, for a given critical speed, a stiffness increase of nearly two to one. However, it is well known that it is mainly in the outer sections of a wing (which suffer far bigger displacements than the root sections) that the chordwise distribution of mass is potent in affecting the flutter problem. An indication of this may be provided by the work of Minhinnick and Yarwood[21] on the flutter of a wing carrying concentrated masses (engine and undercarriage) near the root. Additions of mass corresponding to about one-tenth of the engine mass produce no significant change in the critical flutter speed of the bare wing, though the change in the position of the local centre of mass over the inner third of the wing span is very large.

Accordingly, it appears that in equation (8) it is sufficient if the value of g is determined by the chordwise position of the centre of mass averaged over the outer half of the semispan only. It may be remarked that from the flutter viewpoint it is extremely advantageous to arrange the distribution of mass in the outer half of the wing to be such that the centre of gravity is as far forward as possible.

5 The Proposed New Criterion

In view of the remarks in Para.4 and of the expression for the critical flutter speed given by equation (8), the following torsional stiffness criterion is tentatively proposed:

$$\frac{1}{V}\left(\frac{m_\theta}{dc^2_m}\right)^{\frac{1}{2}} > C\ \frac{(g - 0.1)\ \phi\ (M)}{(1 - 0.8k + 0.4k^2)} \quad .... (9)$$

with $\phi(M) = (1 - M^2)^{-\frac{1}{4}}$, $0 < M < 0.8$

$= 1.29$, $0.8 < M$

and subject to the limitations

$$0.35 < g < 0.55$$

$$0.25 < k < 1.0$$

The value of the constant C would best be determined by a study of values of C appropriate to aircraft which have flown; but a probable value will be about 0.06. This would give a ratio of critical speed to maximum diving speed of about 1.3. It is relevant here to compare equation (8) with the inequality (9). It will be observed that the compressibility correction ϕ (M) has been introduced, but the factors involving h and r have been omitted. This omission is made for two reasons. First, within practical ranges of variation (usually $0.2 < h < 0.4$ and $\frac{1}{2} < r < 2$) the effects of the factors are not great, at least by comparison with the effects due to the extremes of change in g and k. Secondly, it is not usually possible to estimate h and r in the early stages of design; indeed, so far as r is concerned, it cannot be evaluated until the left-hand side of (9) is known.

Accordingly, it is proposed that the factors involving h and r should in general be omitted; they might however, be re-introduced when a borderline case is under consideration from the point of view of the granting of a concession.

6 Further Developments and Comments

The foregoing suggestions have so far related to wings not carrying wing engines. The state of research on the flutter of wings carrying wing engines is not such that it is possible at present to formulate any criterion of a simplicity corresponding to (9). However, there seems to be no reason to doubt that the factors depending on g, k and M will be much the same in the case of a wing with wing engines as for a plain wing; probably the former case should include some new factor depending on the location and mass of the engines. At the present stage however, it seems that the best that can be done is to follow the precedent of the existing criterion and use (9) with a different value for the constant C. As a check, it may be worth while to apply equation (8) to that part of the wing outboard of the outermost engine in order to obtain an approximate estimate of the critical flutter speed. The speed obtained in this manner should of course be modified by the application of the compressibility correction.

A development which may be anticipated relates to the method of determination of the stiffness m_θ. At present this is appropriate to symmetrical applied torque. Research at present in hand suggests however, that, when bodily freedoms of the aircraft are taken into account, the antisymmetrical flutter condition may appear first, and in this case the antisymmetrical value of the stiffness m_θ would be required in the criterion.

Again it is to be remarked that the proposed criterion (9) has been developed on almost purely theoretical grounds, and it is obviously highly desirable to obtain some experimental evidence of its validity as a preventive of flexure-torsion flutter. To this end, a family of model wings of equal span and mean chord, but having different tapers is being constructed and is to be subjected to wind tunnel tests: it will be possible to vary the parameter g in all cases. In this way it should be possible to obtain a check on the factors involving g and k in (9) and an indication of the appropriate value of the constant C.

A problem of current interest is that of the stiffness of swept-back wings. It must be emphasised that the criterion (9) as developed relates to unswept wings, and research into the flutter of swept wings will be necessary before it can be assumed that the criterion (9) is adequate for swept wings or whether (as is probable) it requires modification. In any case, it seems certain that the stiffness of swept wings will depend acutely on the problems of lateral control and longitudinal stability.

One further remark is desirable here. The criterion (9), in common with all existing stiffness criteria, is not non-dimensional, and accordingly the numerical values are not applicable, for example, to the metric system. The criteria could be made non-dimensional by multiplication by $\rho_0^{-\frac{1}{2}}$ i.e. by the factor 20.5 in the numerical value required; the additional factor $\rho^{\frac{1}{2}}$ would then appear with V in the denominator of the criterion and V would be a true speed. If it were thought desirable to make such a modification it would probably be best to square the criterion (and all others) and to express it in a form such as

$$\frac{m_\theta}{\rho V^2 d c_m^2} > X$$

where X is the non-dimensional factor appropriate to the case. This course would give the value of the required stiffness directly, and would avoid the somewhat arbitrary necessity of evaluating the fourth root of $(1 - M^2)$ in the compressibility correction. At the same time it might be worth while to replace in the wing stiffness criterion the spanwise dimension d by the semi-span s, since d has no apparent advantage and is not quite so readily computed.

List of Principal Symbols

B	Critical speed parameter defined in Para.4.1.
C	A constant defined by the inequality (1) also the current stiffness per unit length in the appendix.
C_o	Stiffness per unit length - value at wing root.
I	Current value of torsional moment of inertia per unit length.
I_o	Value of I at the wing root.
M	Mach number.
U	Strain energy.
U_s	Energy in static mode.
U_T	Energy in dynamic mode.
V	Airspeed - usually an equivalent airspeed, except where associated with ρ.
V_c	Critical flutter speed.
c	Current value of wing chord.
c_o	Root value of c.
c_t	Tip value of c.
c_m	Mean value of c.
c'	Value of c at reference section.
d	Distance from wing root to equivalent tip.
g	Defines position of inertia axis aft of wing leading edge.
h	Defines position of flexural axis aft of leading edge.
k	Ratio of tip chord to root chord.
ℓ_ϕ	Flexural stiffness as measured at reference section.
m_θ	Torsional stiffness as measured at reference section.
m^1_θ	Effective value of m in flutter.
n	Torsional frequency defined in relation to equation (4), also index of flutter mode defined by equation. (A.13).
p	Fundamental wing torsional frequency.
r	Stiffness ratio $= \left(\dfrac{\ell_\phi}{d^3}\right) / \left(\dfrac{m_\theta}{dc^2_m}\right)$
s	Semi-span.

List of Principal Symbols (Contd)

y	Spanwise co-ordinate.
α, β	Constants depending on taper given by equation (A.11).
η	Non-dimensional spanwise co-ordinate (= y/s).
η_o	Value of η at reference section.
θ	Current value of torsional amplitude.
μ, ξ, ϕ	Functions defined in Para.2.12 of the appendix.
ρ	Air density.
ρ_o	Value of ρ at sea level.
σ_w	Wing density.
τ	Function of taper (= $1 - k$).

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APPENDIX

The effect of taper on wing flexure torsion flutter speeds

by

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1 Introduction

In the main text of the present report reference has been made to the work of Duncan and Griffith[12], and Buxton[13], both of whom include the effect of taper on wing flutter speeds in their researches. The present investigation was prompted by the consideration that the modes of vibration of a wing are likely to vary with its taper and thereby contribute to the overall effect on the flutter speed.

The flutter problem is approached in the standard way by the assumption of two semi-rigid modes, one of pure flexure and the other of torsion about the flexural axis. It has been shown by Jahn and Buxton[14] that variations in the flexural mode are of much less importance than variations in the torsional mode, so for simplicity the flexural mode is kept constant for the different tapers. The semi-rigid torsional mode is assumed similar to the fundamental torsional mode of the wing vibrating in vacuo; the latter mode is first calculated for each wing and an approximation to it, more suitable for use in the flutter calculation, is then adopted.

The results obtained in the present investigation show some difference from those of Refs.12 and 15, but the effect of the different modes is not great; such as it is, however, the variation of critical speed with taper is reduced. But these results all refer to wings having the same torsional stiffness as measured in a stiffness test at the reference section (0.7 s from the root). That this stiffness is less than the effective stiffness in the actual torsional mode of vibration has been recognised for some time, but previously, so far as the writers are aware, no attempt has been made to estimate how the magnitude of this difference depends on the static and dynamic torsional modes and on the location of the reference section. The effect is in fact considerable. It is discussed in Para.3 of this appendix, and it is shown that the effective stiffness of a strongly tapered wing is relatively much greater (for the standard reference section) than that of an untapered or slightly tapered wing.

Finally a brief enquiry is made into the question of determining the best linear taper to use as an approximation for wings whose plan form is not straight tapered. One or two variations of an approach depending on the least squares principle are tried, but it is concluded that Hirst's conventional method is the most satisfactory.

2 Variation of the Torsional Mode with Taper

The calculations are based on the assumption that the torsional mode under the flutter conditions is not very different from the fundamental torsional mode of vibration in the absence of air forces. That good justification exists for this assumption has been demonstrated in R & M 1716[22]. In Fig.8 of that report the two modes are directly compared, and if the amplitudes are made to coincide at the reference section ($\eta = 0.7$) instead of the tip, the only appreciable discrepancy is at the tip where the flutter mode rises more sharply.

In so far as the present investigation is concerned, this effect is to some extent compensated by use of the approximate modes of Para.2.2 in this Appendix.

2.1 The Exact Solution for the Fundamental Torsional Mode

2.11 Definitions

The geometrical assumptions made are shown in Fig.3. Four different tapers are considered such that

$$c_t/c_o = 1,\ 0.75,\ 0.5 \text{ and } 0.25.$$

Finally it is assumed that the local values both of stiffness and inertia are proportional to the fourth power of the chord.

These assumptions may be expressed in the form

$$\left.\begin{aligned} C &= C_o\,(c/c_o)^4 \\ I &= I_o\,(c/c_o)^4 \end{aligned}\right\} \qquad (A.1)$$

where

C is the local stiffness and C_o the root value

and

I is the local moment of inertia about the flexural axis*, and I_o is the root value.

For convenience we define the taper by means of a quantity τ such that

$$c = c_o\,(1 - \tau\eta) \qquad (A.2)$$

where $\eta = y/s$ is a non-dimensional spanwise variable.

The equations (A.1) may now be written

$$\left.\begin{aligned} C &= C_o\,(1 - \tau\eta)^4 \\ I &= I_o\,(1 - \tau\eta)^4 \end{aligned}\right\} \qquad (A.3)$$

2.12 Solution of the Differential Equation for Torsional Oscillations

The method adopted follows closely that given by Walker[23]. The differential equation for torsional oscillations of the wing may be written

$$I\ddot{\theta} - \frac{\partial}{\partial y}\left(C\,\frac{\partial \theta}{\partial y}\right) = 0 \qquad (A.4)$$

For the fundamental mode of vibration, moreover

$$\theta = \theta_1 \sin(pt + \epsilon)$$

$$\text{or } \ddot{\theta} = -p^2\theta$$

* In the flutter calculations, the moment of inertia about the axis of inertia is assumed proportional to the fourth power of the chord, with the same constant of proportionality appropriate to the different wings. However, the mass per unit span is assumed proportional to the square of the chord so that for a given wing the equation (A1) still holds about the flexural axis.

Whence, substituting in (A.4) and changing the variables we have

$$\frac{1}{s^2}\ \frac{d}{d\eta}\left\{C_o\,(1 - \tau\eta)^4\ \frac{d\theta}{d\eta}\right\} + p^2\, I_o\,(1 - \tau\eta)^4\theta = 0 \qquad (A.5)$$

by use of the equation (A.3)

The equation (A.5) is now modified to a form of Bessel's equation. It may readily be verified that by making the substitutions

$$\left.\begin{aligned} 1 - \tau\eta &= \tau\,\xi \\ \text{and}\quad \theta &= \phi\xi^{-3/2} \end{aligned}\right\}$$

the equation reduces to

$$\mu^2\,\frac{d^2\phi}{d\mu^2} + \frac{\mu d\phi}{d\mu} + \left\{\mu^2 - \left(\frac{3}{2}\right)^2\right\}\phi = 0 \qquad (A.6)$$

where

$$\mu^2 = \xi^2\, s^2\, p^2\, I_o/C_o.$$

The solution of equation (A.6) is

$$\phi = A\,J_{3/2}\,(\mu) + B\,J_{-3/2}\,(\mu) \qquad (A.7)$$

where A and B are arbitrary constants and J refers to the appropriate Bessel function. In this case the Bessel functions may be written as simple trigonometrical expressions, and the solution is accordingly

$$\mu^{3/2}\phi = A(\sin\mu - \mu\cos\mu) + B(-\mu\sin\mu - \cos\mu) \qquad (A.8)$$

This equation must satisfy the end conditions that

$$\left.\begin{aligned} \theta &= 0 \text{ when } \eta = 0 \\ \text{and}\quad \frac{d\theta}{d\eta} &= 0 \text{ when } \eta = 1 \end{aligned}\right\} \qquad (A.9)$$

The substitution of conditions (A.9) in equation (A.8) yields two equations from which a relation between A and B is obtained, thus specifying the mode of vibration; a second relation between the relevant parameters is also obtained which will give the frequency. After carrying out this process, the resulting equations may be written

$$\beta^{\frac{1}{2}}\,(1 - \tau\eta)^2\left(\frac{\theta}{A}\right) = \left\{\frac{\sin[\beta\,(1 - \tau\eta)]}{\beta\,(1 - \tau\eta)} - \cos[\beta\,(1 - \tau\eta)]\right\}$$

$$- \alpha\left\{\sin[\beta\,(1 - \tau\eta)] + \frac{\cos[\beta\,(1 - \tau\eta)]}{\beta\,(1 - \tau\eta)}\right\} \qquad (A.10)$$

where

$$\left.\begin{aligned} \alpha &= \frac{\beta\cos\beta - \sin\beta}{\beta\sin\beta + \cos\beta} \\ \text{and}\quad \tan\tau\beta &= \frac{\beta\{3\tau - \beta^2(1-\tau)^2\}}{3 + \beta^2(2+\tau)(1-\tau)} \end{aligned}\right\} \qquad (A.11)$$

The quantity β is defined as the positive root of the expression

$$\beta^2 = \frac{s^2 p^2}{\tau^2} \frac{I_o}{C_o}$$

whence the frequency p is given by

$$p = \frac{\tau\beta}{s} \left(\frac{C_o}{I_o}\right)^{\frac{1}{2}} \qquad (A.12)$$

Thus for a prescribed value of τ, the parameter β is evaluated from the second of equations (A.11), and hence the mode of vibration is obtained from (A.10) and the frequency from (A.12). The value of β required is the lowest positive value given by equation (A.11).

2.2 An Approximate Mode used in the Flutter Calculations

The modes of vibration as determined in Para.2.1 could be used directly in the flutter calculations. The complex analytical form of the expressions for the modes, however, would necessitate a graphical method for the evaluation of the flutter integrals, and in view of the numerical accuracy required for these values, it was decided to approximate to the modes found, so as to give integrals which may be simply evaluated analytically. Forthis purpose the approximate modes are found in the form

$$f = A_1 \eta^n \qquad (A.13)$$

where f is the torsional mode and A_1 is a constant.

The torsional mode given by (A.13) is obtained by the principle of least squares from the exact mode found above. This leads to the results that

$$\left.\begin{aligned} &\Sigma f \eta^n - \Sigma A_1 \eta^{2n} = 0 \\ \text{and} \quad &\Sigma(f A_1 \eta^n \log\eta) - \Sigma(A_1^2 \eta^{2n} \log\eta) = 0 \end{aligned}\right\} \qquad (A.14)$$

Elimination of A_1 from equation (A.14) yields

$$\Sigma(f \eta^n \log\eta) \;\; \Sigma\eta^{2n} = \Sigma(\eta^{2n} \log\eta) \; \Sigma f \eta^n \qquad (A.15)$$

from which the value of n is deduced by a process of trial and error.

The summations appearing in (A.14) and (A.15) are strictly integrals taken along the span. However, it was convenient to effect the integrations by taking summations of finite spanwise increments. In this way, Equation (A.15) has been solved for the four values of τ considered and by applying the least sqares principle to ten increments of η. The exact and approximate modes are compared in Figures 4 - 7. It should be mentioned that the ordinates of these graphs have no special significance, and for the flutter calculations the value is standardised as unity at the reference section. The value of the ordinate at the reference section is not, however, assumed equal to that in the exact mode (as may be seen by equations (A.14)) for the calculations of the approximate mode, as this would impose an additional restraint on the mode.

It is interesting to observe that the index n increases with taper and is about unity for an average taper of $\tau = 0.5$. For the condition $\tau = 0$ it may be seen that $\beta = \pi/2$ (from equation (A.11)) which leads to the well known result that the exact mode is a sine curve, and in the approximation n is about 0.7. Table 1 gives the variations of the index n with taper.

Table 1

τ	$k = \frac{\text{tip chord}}{\text{root chord}}$	n
0	1	0.680
0.25	3/4	0.835
0.5	1/2	0.968
0.75	1/4	1.079

3 The variation of effective stiffness with taper

The investigations carried out under the above heading have proved to be among the most important, quantitatively, in the development of the new criterion proposed in the main text. The whole of the effect may be related to two defects in the semi-rigid hypothesis adopted in flutter work; namely that the predicted critical speeds will in fact depend on the reference section chosen, and on the relation between the effective stiffness in the mode of vibration and the stiffness which would normally be measured by a stiffness test. In this paragraph an attempt, based on the assumption of Para.2.11, is made to compensate for these defects.

3.1 The Choice of a Reference Section

This question has been considered by Victory[16]. For one particular plan form, she calculated the critical flutter conditions with different assumed modes of vibration, and concluded that variations in critical speed were minimised if the reference section were chosen at about 0.7s from the root. If therefore the modes for any given wing were not known, the probable error in critical speed would be least if this reference section were adopted.

In the present investigation, as detailed in the following paragraph the relation between the measured and effective stiffness is examined for each taper, and for a range of values of the reference section. It was hoped that the reference section giving the peak value of $(m_\theta / m_\theta{}^1)$ where m_θ is the measured and $m_\theta{}^1$ the effective stiffness, could be chosen as standard, and that if this peak were about constant and not far from unity the measured stiffness could be used directly in the flutter work. The results, however, show widely different characteristics for the different tapers, as may be seen in Fig.8, where the ratio $(m_\theta / m_\theta{}^1)$ is plotted against the value of η at the reference section. It is evident that an effective stiffness allowance must be made; and from this consideration, and from the positions of the maxima shown by the graphs, there is clearly no case for changing the reference section from the standard one of $\eta_0 = 0.7$.

3.2 The Effective Stiffness for Different Reference Sections and Different Tapers

The problem is approached by the consideration of the energies in the two relevant modes; the one mode is that obtained by the method of Para.2, and the other is that appropriate to a concentrated torque applied at the reference section. The dynamical mode adopted is the exact mode derived for the fundamental torsional oscillations, and not the approximate mode used in the flutter calculations. The reason for this procedure is that the flutter mode is purely a geometrical representation of the exact mode for aerodynamical purposes, and was derived so as to have as little effect as possible on the critical speed which would be appropriate to the exact mode. The stiffness of the approximate mode is not, therefore, relevant to the problem, and in fact it may be noted that the root condition of torsional oscillations is not satisfied.

The problem of the untapered wing is considered first; the solution for the tapered wings is the same in principle but algebraically more complicated.

3.21 The Untapered wing

The strain energy in the wing is derived from the general equation

$$dU = \tfrac{1}{2}\,\frac{C}{s}\left(\frac{d\theta}{d\eta}\right)^2 d\eta \tag{A.16}$$

We first consider the case of a static concentrated torque applied at some reference section η_o. As C is constant in the present case, the mode of distortion increases linearly as far as the reference section and is constant beyond that section. If the twist at the reference section is θ_o the energy is given (from A.16) by

$$U_s = \int_0^{\eta_o} \cdot \tfrac{1}{2}\,\frac{C}{s}\cdot\left(\frac{\theta_o}{\eta_o}\right)^2 d\eta$$

$$= \tfrac{1}{2}\,\frac{C}{s}\,\frac{\theta_o^{\,2}}{\eta_o} \tag{A.17}$$

For the dynamic mode θ is proportional to $\sin\frac{\pi}{2}\eta$, and since, by definition, $\theta = \theta_o$ when $\eta = \eta_o$, we have

$$\theta = \theta_o\,\frac{\sin\frac{\pi}{2}\eta}{\sin\frac{\pi}{2}\eta_o} \tag{A.18}$$

Whence, again by (A.16)

$$U_T = \tfrac{1}{2}\,\frac{C}{s}\cdot\frac{\pi^2\,\theta_o^{\,2}}{4\sin^2 \pi/2\ \eta_o}\int_0^1 \cos^2(\tfrac{1}{2}\pi\eta)\,d\eta$$

or

$$U_T = \frac{\pi^2\,C\theta_o^{\,2}}{16\,s\,\sin^2\frac{\pi\eta_o}{2}}$$

The ratio of energies is obtained from equations (A.17) and (A.19).

$$\frac{U_s}{U_T} = \frac{8}{\pi^2 \eta_o} \sin^2\left(\frac{\pi\eta_o}{2}\right)$$

Moreover, as the twist at the reference section is the same in the two modes, this ratio is also the ratio of the measured stiffness to the effective stiffness. Whence

$$\frac{m_\theta}{m_\theta^1} = \frac{8}{\pi^2 \eta_o} \sin^2\left(\frac{\pi\eta_o}{2}\right) \qquad \text{(A.20)}$$

The ratio (m_θ / m_θ^1) is plotted in Fig.8 against η_o from equation (A.20). It can be seen from this graph, or obtained analytically, that the peak occurs at $\eta_o = 0.74$ approximately, and for this condition the ratio is a little more than 92%. For η_o unity, moreover, the result agrees with that obtained by Duncan and Collar in R & M. 1518[11]. In Fig.10 the static and dynamic modes are compared for the standard reference section, and this comparison does suggest that the static mode resembles the dynamic mode most closely for a reference section at a value of η of about 0.7.

3.22 The Tapered Wing

The static mode is no longer linear, and is given by

$$\theta \propto \frac{1}{C_o} \int_o^\eta \frac{d\eta}{(1 - \tau\eta)^4}$$

or
$$\theta \propto \frac{1}{3\tau C_o} \left\{ \frac{1}{(1 - \tau\eta)^3} - 1 \right\}$$

whence

$$\theta = \frac{\theta_o (1 - \tau\eta_o)^3}{1 - (1 - \tau\eta_o)^3} \cdot \frac{1 - (1 - \tau\eta)^3}{(1 - \tau\eta)^3}$$

and

$$\frac{d\theta}{d\eta} = \frac{\theta_o (1 - \tau\eta_o)^3}{1 - (1 - \tau\eta_o)^3} \cdot \frac{3\tau}{(1 - \tau\eta)^4}$$

The energy in the static mode is therefore (by (A.16))

$$U_s = \frac{C_o}{2s} \left\{ \frac{\theta_o (1 - \tau\eta_o)^3}{1 - (1 - \tau\eta_o)^3} \right\}^2 \int_o^{\eta_o} \frac{9\tau^2}{(1 - \tau\eta)^4} d\eta$$

$$= \frac{C_o}{2s} \left\{ \frac{(1 - \tau\eta_o)^3}{1 - (1 - \tau\eta_o)^3} \right\} \cdot 3\tau\theta_o^2$$

To determine the energy in the dynamic mode U_T, $\frac{d\theta}{d\eta}$ is found by differentiation of (A.10). The result is so complicated algebraically that the integrations for U_T have been carried out graphically (by use of Simpson's rule). The ratio U_s/U_T or $m_\theta / m_\theta{}^1$ is then obtained directly as before.

The graphs now obtained for $m_\theta / m_\theta{}^1$ are plotted against the reference section in Fig.8. These graphs show that the peak of the curve moves towards the root as the taper becomes more pronounced. At the same time the ordinate at the maximum increases with taper while the taper is small, but as the taper becomes more pronounced it starts to fall off again. For high tapers this falling off of the maximum together with its inboard trend causes a pronounced variation in the ratio $m_\theta / m_\theta{}^1$ for the standard reference section ($\eta_o = 0.7$) such that as τ varies from $\frac{1}{4}$ to $\frac{3}{4}$ the ratio drops from 0.97 to 0.69. The variation with τ for $\eta_o = 0.7$ is given in Fig.9.

For the purpose of comparison with Fig.10 which applies to the untapered wing, the static and dynamic modes are shown in Fig.11 for the standard reference section when $\tau = 0.75$. The disparity between the two modes is greater in the case of the strongly tapered wing, as would be expected from the energy difference.

3.3 Relation to the Criterion

As the criterion proposed is primarily a flutter criterion, the torsional stiffness on which it depends should be that appropriate to the flutter mode - the effective stiffness m'_θ. This stiffness, however, expressed in terms of the stiffness as normally measured in a static test, depends only on the taper with the assumptions made in the present report and for a standard reference section. For obvious reasons it is more convenient that the stiffness put into the criterion should be the normal test value, and accordingly the variation with τ given by Fig.9 is incorporated in the overall variation with taper, and is implicit in the right hand side of equation (8) and the relation (9) of the main text.

4 The Flutter Calculations

The basis of the flutter calculations is the method described in R. & M. 1782[10]. The same binary approach is followed, and apart from the exceptions given below the same assumptions are made. In those circumstances it has not been thought worth while to reproduce the flutter equations here; in the main they are identical with those given in R. & M. 1782, and the slight modifications necessary to the various integrals are quite obvious. One point of similarity, however, deserves special mention. The aerodynamic derivatives used are the constant derivatives first introduced by Duncan and Collar[11], and adopted in many later works (Refs.10 and 12 for example). These derivatives do not vary with taper, and thereby introduce an assumption which may affect the variation of critical speed with taper, and which has not been justified experimentally. For although derivatives have been measured on both tapered and untapered wings (the results are given in two papers, 24, 25 by Williams) the results available are not sufficiently systematic or complete for a variation with taper to be deduced. The experiments mentioned in Para.6 of the main text may provide some valuable information on this question.

The differences between the present calculations and those of R. & M.1782 may be summarised as

(i) The torsional mode is varied with taper as described in Para.2.

(ii) The flexural mode is assumed parabolic.

(iii) The sectional moments of inertia are assumed constant (for a given chord) about the axis of inertia. For the terms involving this moment of inertia in the flutter equations (the γ terms) a correction is applied to give the value appropriate to rotation about the flexural axis.

(iv) The solution in terms of m_θ^1 is connected to a solution in terms of m_θ by the method of Para.3.

Some brief comments on these points may perhaps be worth while. Points (i) and (iv) are closely related and have already been discussed in some detail. The assumption (ii) is made chiefly for the purpose of simplifying the evaluation of the integrals. The flexural mode adopted in R. & M.1782[10] (and later in R. & M.1869[12]) is an empirical one and would necessitate graphical treatment, whereas a mode of the form $f = k\eta^n$ is simply integrated analytically. The mode of R. & M.1782 corresponds roughly to a value of n of 1.8, but the simple parabola is thought to be quite as typical of the modes likely to be experienced in practice. Point (iii) above is made to correct a slight inconsistency in R.& M.1782 where the moments of inertia about the flexural axis, and not about the axis of inertia are constant; this was pointed out by Buxton[15].

The present investigation includes the variation of several parameters. These are indicated by Table II.

Table II

	h = 0.3	h = 0.35
τ	0, 0.25, 0.5, 0.75	0, 0.25, 0.5, 0.75
g	0.4, 0.45, 0.5	0.4, 0.45, 0.5
r	0, 0.5, 1.0, 1.5	1.0

The value of σ_W adopted throughout is 1.6 pounds per cubic foot. The results of the main part of the investigation, given by the left hand side of Table II, are shown in Fig.2, (where $k = 1 - \tau$). To complete the picture, the three graphs of B against τ with different values of g are plotted for each value of h in Fig.12. In all these graphs the effective stiffness m_θ^1 has been replaced by the measured stiffness m_θ by the method of the preceding paragraph. It will be noted that the effect of h alone is small - especially so when the separation between the flexural and inertial axes is large.

5 Definition of Taper in Non-linear cases

The problem of defining the taper of a wing, which is not straight tapered has always been difficult. Clearly it is not sufficient in such a case to take the ratio of the actual tip chord to the actual root chord; an elliptic wing, for example, could not be represented in this manner.

There are two standard ways of defining taper, and three new methods are considered here. The five methods will, for convenience, be referred to by numbers. First, the standard methods.

(1) Consider a straight tapered wing with the same area, the same span and same root chord as the actual wing.

(2) The same as (1) but with constant mid-aileron chord instead of root chord.

The first method is that used for stressing problems. The second is that of Hirst[26] used in aileron reversal work; here we adopt the section $\eta = 0.7$ (the reference section) rather than the mid-aileron section.

The new methods are based on the principle of least squares. In each case an equivalent wing is considered whose taper is given by

$$\text{chord} = a + b\,\eta$$

where a and b are constants. If c is the local chord on the actual wing, methods 3, 4 and 5 may be stated

(3) the span is constant and $\Sigma(a + b\eta - c)^2\, d\eta$ is a minimum.

(4) the span is constant and $\Sigma(a + b\eta - c)^2\eta\, d\eta$ is a minimum.

(5) the span is constant and $\Sigma(a + b\eta - c)^2\eta^2\, d\eta$ is a minimum.

Method (3) is the normal least squares result, and methods (4) and (5) involve weighting of the tip sections. It should be noted that method (3) automatically leads to an equivalent wing of the same area as the actual wing. Methods (4) and (5) on the other hand do not, and correction must be made accordingly to the value of c_m used in the criterion.

Comparison of the methods is effected by first obtaining the critical speed of two wings, one of elliptical taper and the other of polygonal shape with zero taper at the root and pronounced taper toward the tip. The former result was obtained from Buxton[15] and the latter worked out with (in both cases) a linear torsion mode and the other assumptions as in Para.4 of this appendix. These results could then be compared, for various conditions, with those given by the different linear tapers as obtained by methods (1) to (5) above.

This comparison has been made, and the conclusion reached was that methods (2) and (5) give the best results, with little to choose between them. Under these circumstances it is obviously desirable that method (2), in view of its extreme simplicity, be adopted as standard. No attempt is made here to give the complete results of this investigation but in Table III a specimen of the results is given which shows the order of variation. The quantity K is the critical value given by the expression (2) of the main text. The appropriate values of K are given under the various conditions indicated.

Method	elliptical wing			polygonal wing		
	$\sigma_w = 0.4$	$\sigma_w = 0.8$	$\sigma_w = \infty$	$\sigma_w = 0.4$	$\sigma_w = 0.8$	$\sigma_w = \infty$
1	0.0167	0.0213	0.0239	0.0165	0.0206	0.0238
2	0.0171	0.0208	0.0242	0.0146	0.0189	0.0225
3	0.0148	0.0190	0.0226	0.0143	0.0188	0.0223
4	0.0142	0.0190	0.0230	0.0136	0.0186	0.0226
5	0.0135	0.0195	0.0239	0.0136	0.0187	0.0229
True Value	0.0154	0.0204	0.0244	0.0136	0.0186	0.0226

In Table III the values of r, h and g are 1.0, 0.3, and 0.4 respectively. It may be noted from Table III that Method (1) breaks down badly for the polygonal wing. A further noteworthy point is that the error of method (2), apart from the low wing density case, is only about 2%.

The taper as defined by method (2) may be expressed in the form

$$k = \frac{c' - 0.6\, c_m}{1.4\, c_m - c'}$$

where c_m is the mean chord, and c' the chord at the section $\eta = 0.7$.

6 Conclusions

Some conclusions of interest may be drawn from the results of this appendix, apart from their application to the proposed wing stiffness criterion. It is shown that the difference between the effective and measured stiffness may be quite large - even for conventional wings. For such cases the effect is thought to be adequately covered by the variation of the criterion with taper; but if detailed flutter calculations are to be made in any specific case (where, perhaps, the margin of safety is critically small or where other features render special calculation advisable) the wing stiffness would best be obtained by measurements (or calculation) in which the wing is made to twist in its assumed vibration mode.

It is also concluded that the equivalent taper of a non-uniform wing may be obtained by a method analogous to that of Hirst[26] for aileron reversal, to within an accuracy of about 2% on critical speed.

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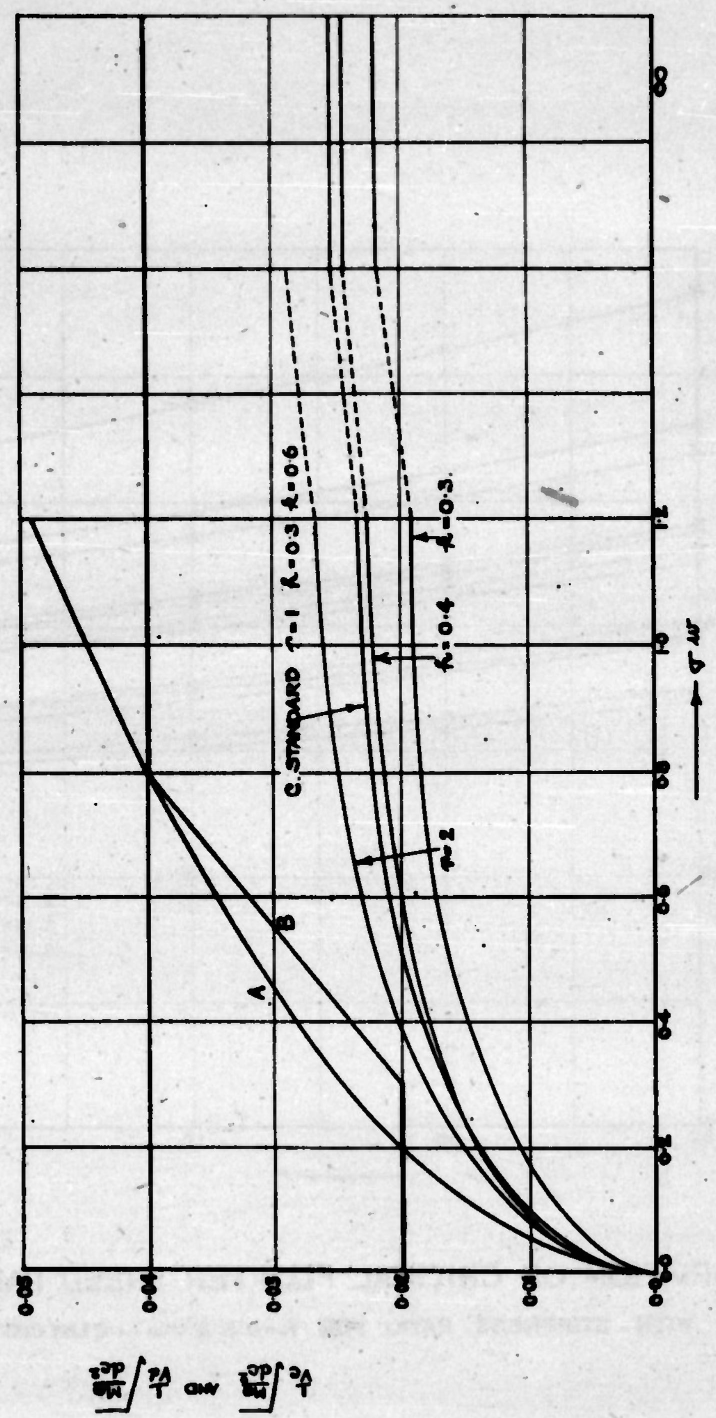
FIG. 1.



VARIATION OF FLUTTER PARAMETERS WITH WING DENSITY.

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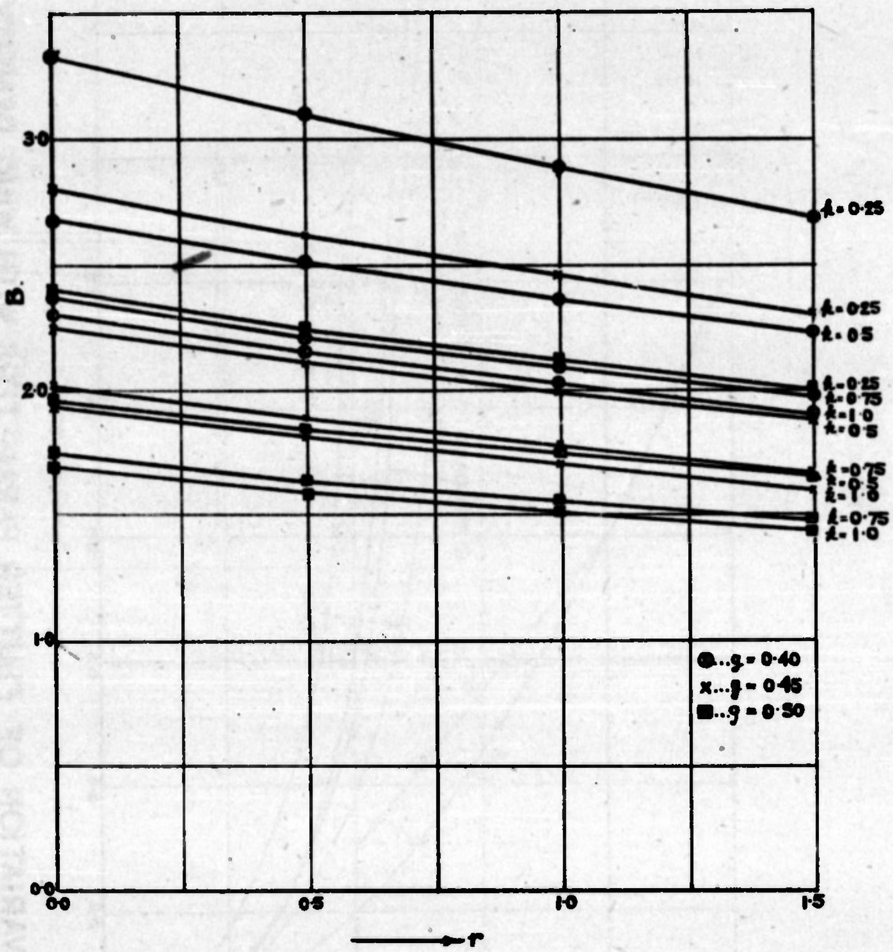
FIG. 2.



VARIATION OF CRITICAL FLUTTER SPEED PARAMETER (B)

WITH STIFFNESS RATIO FOR f_r = 0·3 & σ_w = 1·6 LB./CUB.FT.

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FIG. 3.

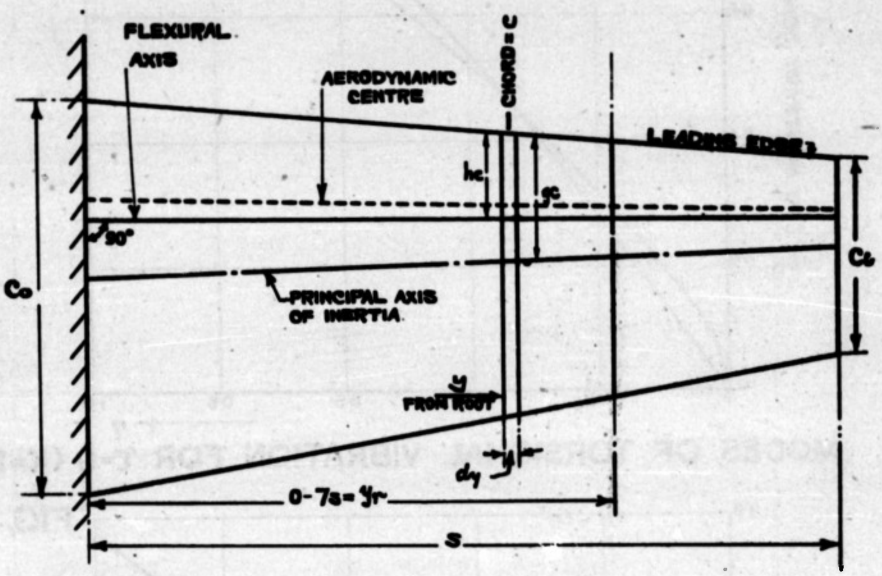

DIAGRAM OF TAPERED WING.

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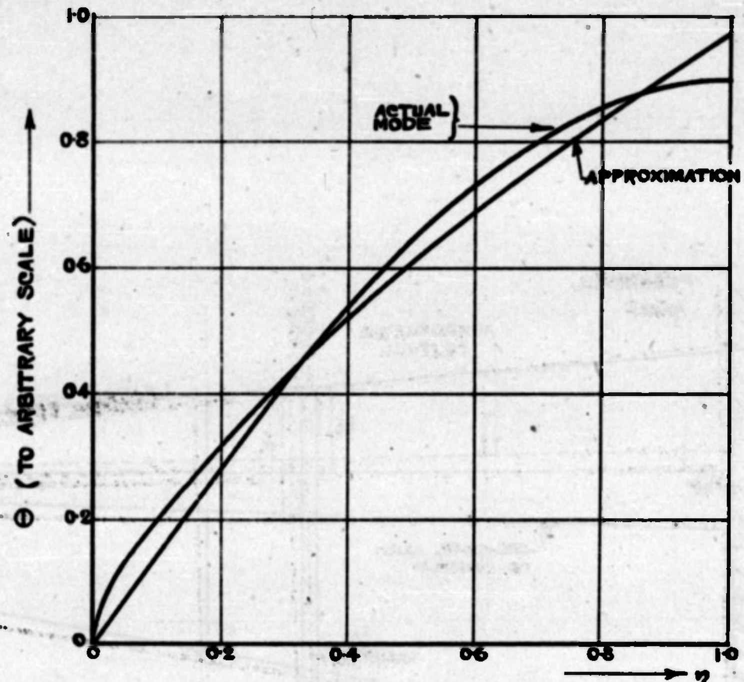
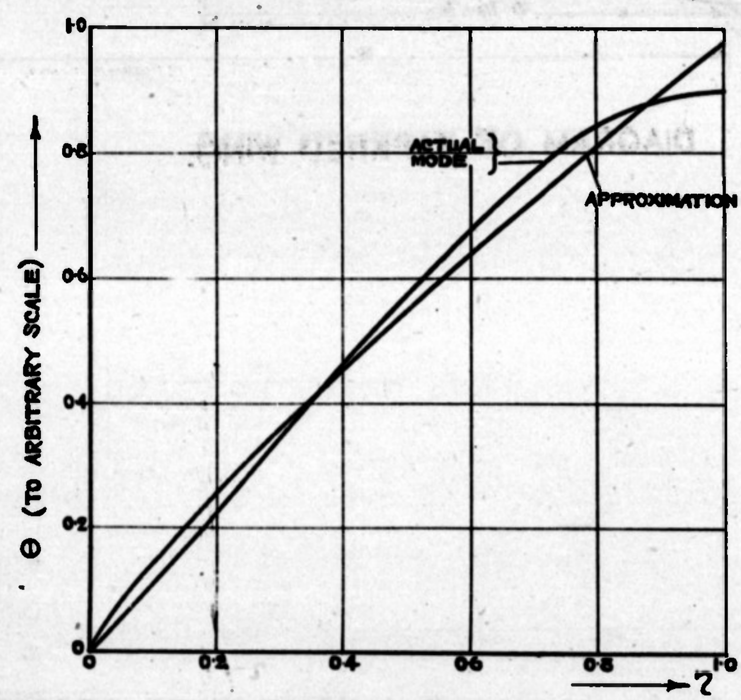
FIGS. 4 & 5.

FIG. 4.



MODES OF TORSIONAL VIBRATION FOR τ-0 (K=1)

FIG. 5.



MODES OF TORSIONAL VIBRATION FOR τ-0·25 (K=¾)

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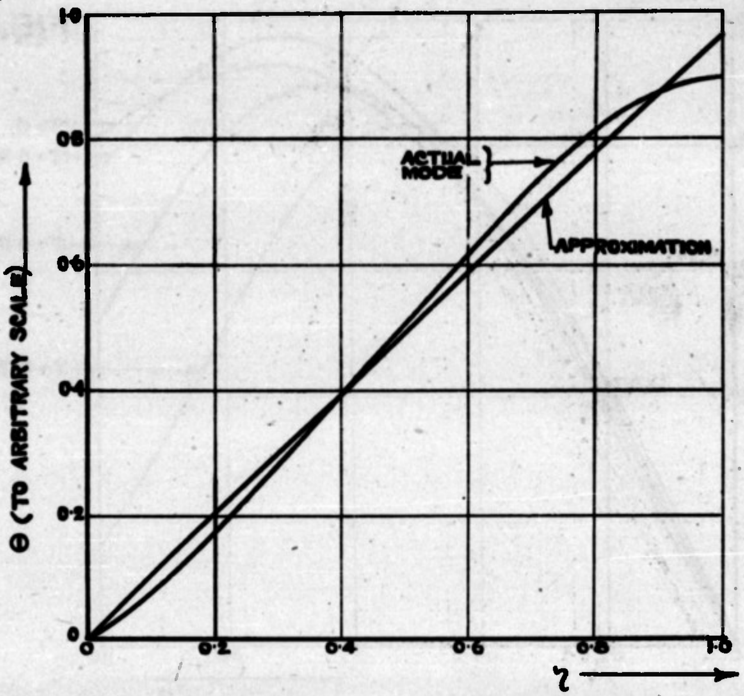
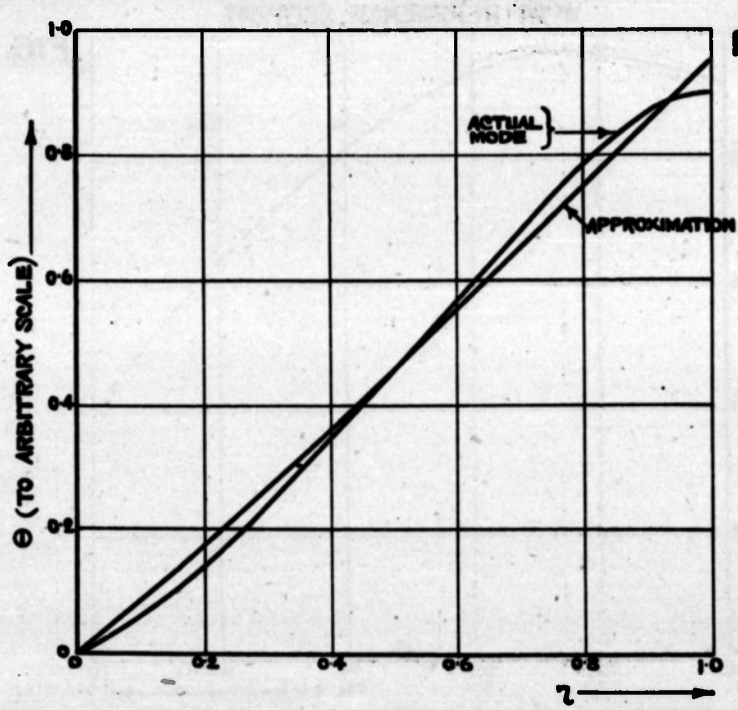
FIGS. 6 & 7.

FIG. 6.



MODES OF TORSIONAL VIBRATION FOR τ = 0·5 (K = ½).

FIG. 7.



MODES OF TORSIONAL VIBRATION FOR τ = 0·75 (K = ¼).

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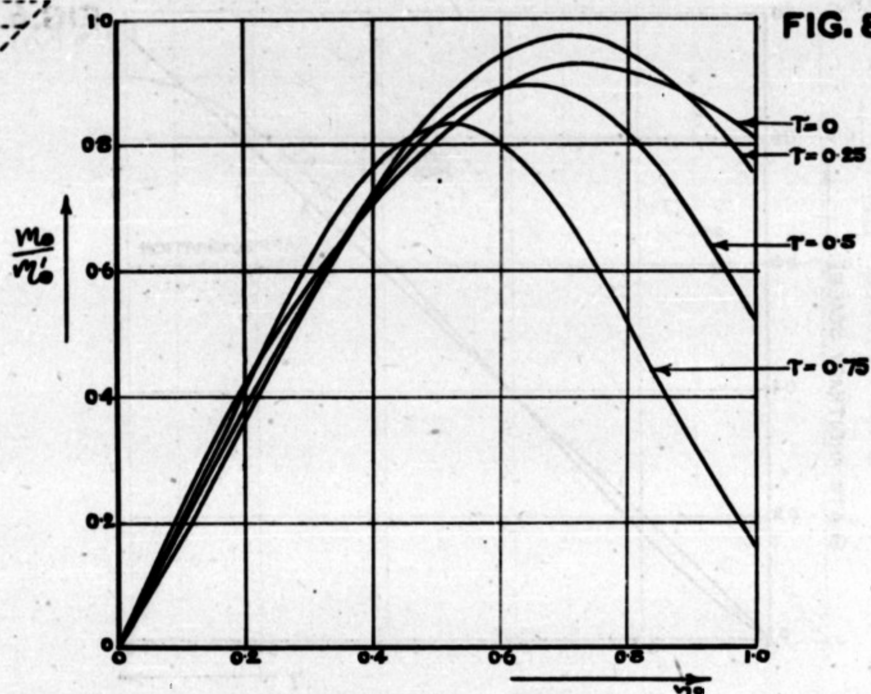
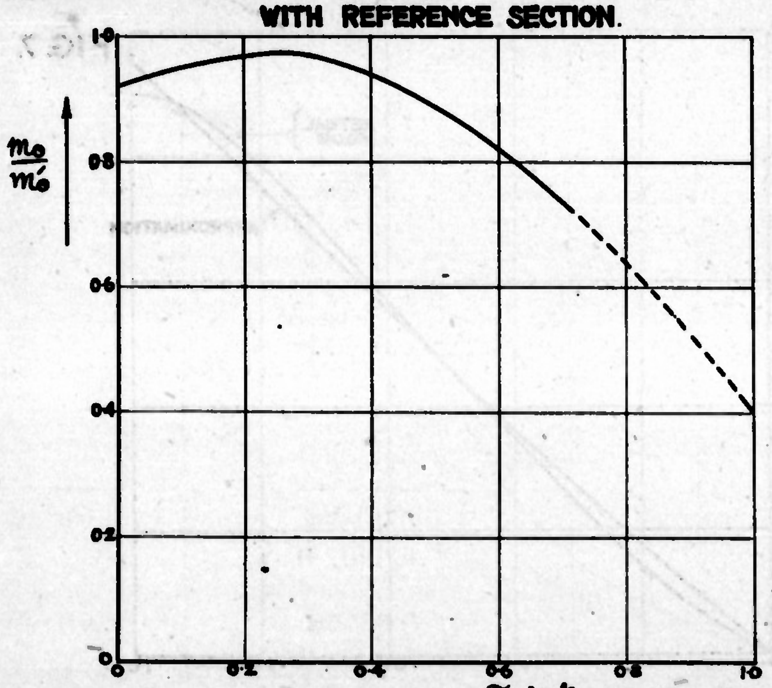
FIG. 8.



VARIATION OF RATIO OF MEASURED STIFFNESS TO EFFECTIVE STIFFNESS WITH REFERENCE SECTION.

FIG. 9.



VARIATION OF RATIO OF MEASURED STIFFNESS TO EFFECTIVE STIFFNESS WITH TAPER FOR $\frac{x}{l_o}$ = 0·7.

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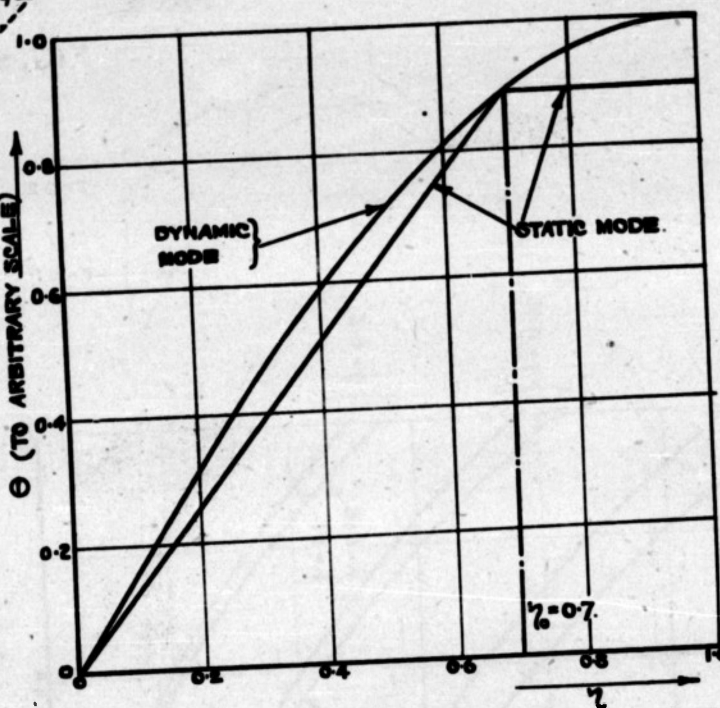
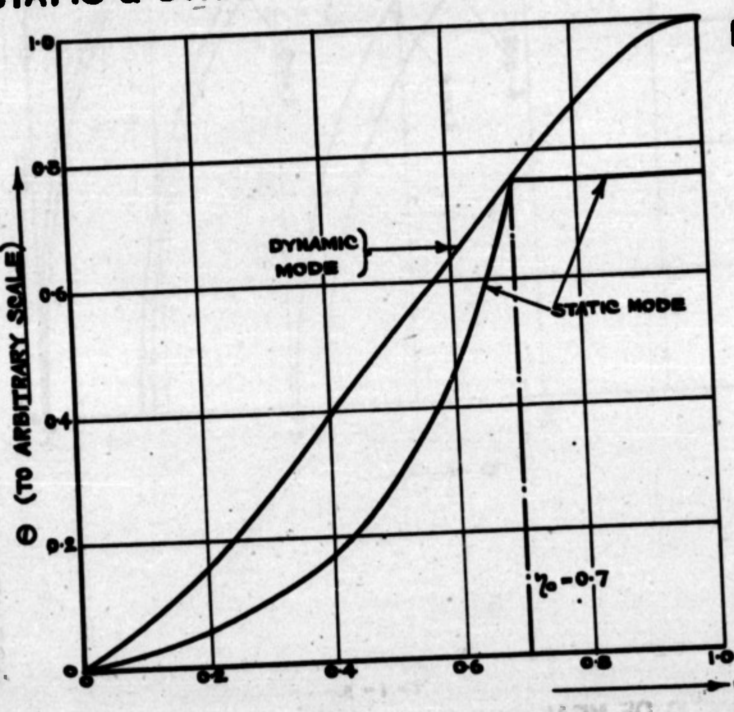
FIGS. 10 & 11.

FIG. 10.



STATIC & DYNAMIC MODES FOR T=0 & η_0=0·7.

FIG. 11.



STATIC & DYNAMIC MODES FOR T=0·75 & η_0=0·7.

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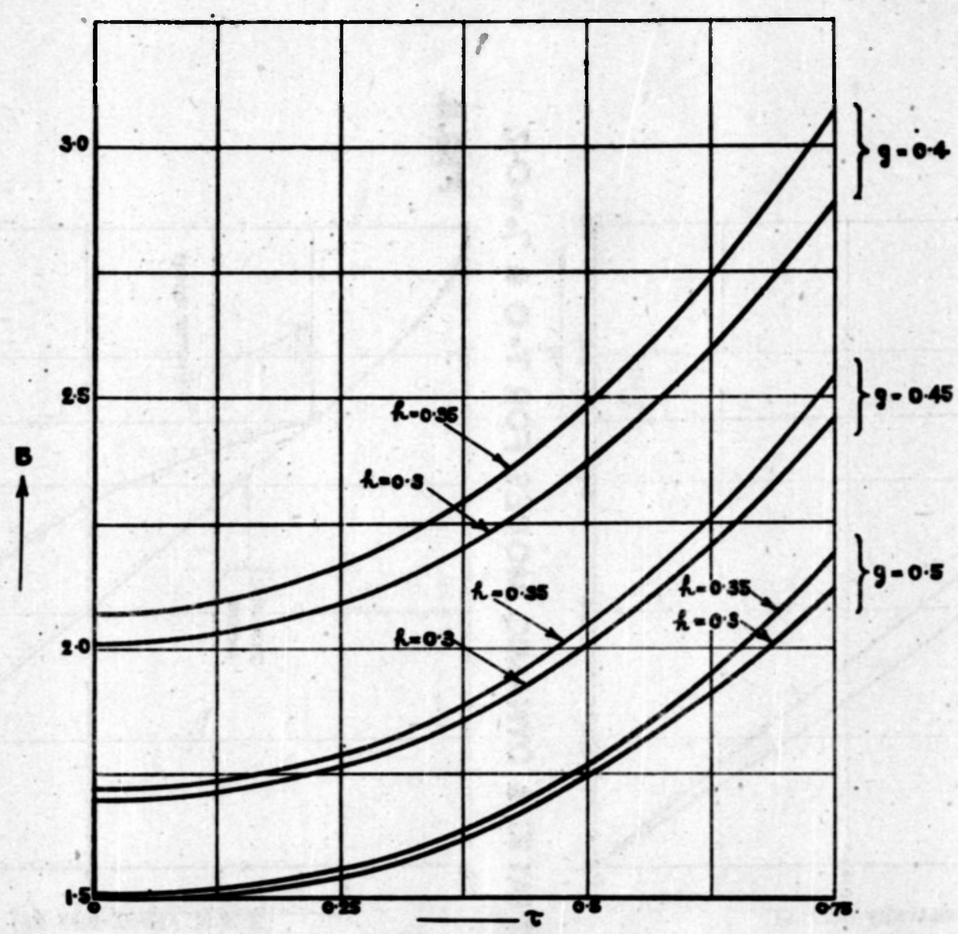
FIG. 12.



VARIATION OF CRITICAL SPEED PARAMETER (B) WITH TAPER FOR $\tau = 1{\cdot}0$ & $\sigma_{w} = 1{\cdot}6$.

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AUTHOR(S): Coelar, A. R.; Broadbent, E. G.; Puttick, Elizabeth B.; and others
ORIGINATING AGENCY: Royal Aircraft Establishment, Farnborough, Hants
PUBLISHED BY: (Same)

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
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